Addendum to "...The Relativistic Covariance of the B$-$ Cyclic Relations" [*Found. Phys. Lett.* **10 (1997) 383-391**]

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In the previous paper we proved that the Evans-Vigier definitions of $B^{(0)}$ and $\mathbf{B}^{(3)}$ may be related *not* with magnetic fields but with a 4-vector field. In the present *Addendum* it is shown that the terms used in the $\mathbf{B}-$ Cyclic theorem proposed by M. Evans and J.-P. Vigier may have various transformation properties with respect to Lorentz transformations. The fact whether the $\mathbf{B}^{(3)}$ field is a part of a bi-vector (which is equivalent to antisymmetric second-rank tensor) or a part of a 4-vector, depends on the phase factors in the definition of positive- and negative- frequency solutions of the $(\mathbf{B}, \mathbf{E})$ transverse field. This is closely connected to our considerations of the Bargmann-Wightman-Wigner (Gelfand-Tsetlin-Sokolik) Constructs and with the Ahluwalia's recent consideration of the phase factor related to gravity. The physical relevance of proposed constructs is discussed.

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In their papers and books [1] Evans and Vigier used the following definition for transverse antisymmetric tensor field:

$$\begin{pmatrix} \mathbf{B}_\perp \\ \mathbf{E}_\perp \end{pmatrix} = \begin{pmatrix} \frac{B^{(0)}}{\sqrt{2}} \begin{pmatrix} +i \\ 1 \\ 0 \end{pmatrix} \\ \frac{E^{(0)}}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \end{pmatrix} e^{i\phi} + \begin{pmatrix} \frac{B^{(0)}}{\sqrt{2}} \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix} \\ \frac{E^{(0)}}{\sqrt{2}} \begin{pmatrix} 1 \\ +i \\ 0 \end{pmatrix} \end{pmatrix} e^{-i\phi} \quad , \tag{1}$$

(see refs. [1,2] for detailed notation). On this basis they defined so-called $\mathbf{B}^{(3)}$ filed and the $\mathbf{B}-$ Cyclic Theorem:

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)\,*} \quad \textit{et cyclic} \; . \tag{2}$$

This theory got great deal of criticism, see, for instance, [3–8]. Particularly, Comay claimed [6] that the $\mathbf{B}^{(3)}$ field is incompatible with the Relativity Theory. I commented this discussion in [2] and suggested that the $\mathbf{B}^{(3)}$ field may be interpreted as a part of 4-vector field functions, see also [9–11]. In the present paper I show that the question of the transformation law for such a kind of the field is not trivial and depends on the phase factors between up- and down- parts of electromagnetic bi-vectors (or between parts of the antisymmetric tensor which is equivalent to the former) corresponding to the positive- and negative- frequency solutions. **The achieved result is that the $\mathbf{B}^{(3)}$ field defined as in** (2) **may be a part of the antisymmetric tensor field**. In this case we encounter unusual configurations of the corresponding transverse $\mathbf{B}$ and $\mathbf{E}$, *but* similar unusual configurations of the antisymmetric tensor field have been considered for a long time [12–18]; they are very well-known to the quantum-field theorists (information mainly from the referee and editors of the paper [11] from Physical Review D); and a similar construct found its sound interpretation in the recent Ahluwalia's paper [19], who is perfectly aware about previous considerations [13,16,17,14,12,1] (cf. references in [19]) and with whom we discussed all this staff during last six years.

I would like to pass now to mathematical details.

In refs. [1] the authors used the transverse solutions of the Maxwell's equations (the formula (1) above) in order to define $\mathbf{B}^{(3)}$. These tranverse solutions can be re-written to the real fields:

$$\begin{pmatrix} \mathbf{B}_\perp \\ \mathbf{E}_\perp \end{pmatrix} = \begin{pmatrix} B^0\sqrt{2} \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} \\ E^0\sqrt{2} \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} \end{pmatrix} \; , \tag{3}$$

which represent the right-polarized radiation ($B^0 = E^0$). Of course, similar formulas exist for left-polarized radiation.

The Lorentz transformation law for antisymmetric tensor field (written in the form of the bi-vector) is:

$$\begin{pmatrix}\mathbf{B}\\ \mathbf{E}\end{pmatrix}' = \Lambda \begin{pmatrix}\mathbf{B}\\ \mathbf{E}\end{pmatrix} = \begin{pmatrix} \gamma + \frac{\gamma^2}{\gamma+1}\left[(\mathbf{S}\cdot\boldsymbol{\beta})^2 - \boldsymbol{\beta}^2\right] & i\gamma(\mathbf{S}\cdot\boldsymbol{\beta})\\ -i\gamma(\mathbf{S}\cdot\boldsymbol{\beta}) & \gamma + \frac{\gamma^2}{\gamma+1}\left[(\mathbf{S}\cdot\boldsymbol{\beta})^2 - \boldsymbol{\beta}^2\right]\end{pmatrix} \begin{pmatrix}\mathbf{B}\\ \mathbf{E}\end{pmatrix} . \qquad (4)$$

It is easy to see that the case considered in ref. [1,2] corresponds to the choice of the field function (operator in the quantized case) in the following form:

$$\begin{pmatrix}\mathbf{B}_\perp\\ \mathbf{E}_\perp\end{pmatrix}' = \Lambda \left\{ \begin{pmatrix}\tilde{\mathbf{B}}^{(1)}\\ \tilde{\mathbf{E}}^{(1)}\end{pmatrix} e^{+i\phi} + \begin{pmatrix}\tilde{\mathbf{B}}^{(2)}\\ \tilde{\mathbf{E}}^{(2)}\end{pmatrix} e^{-i\phi} \right\} = \Lambda \left\{ \begin{pmatrix}\tilde{\mathbf{B}}^{(1)}\\ -i\tilde{\mathbf{B}}^{(1)}\end{pmatrix} e^{+i\phi} + \begin{pmatrix}\tilde{\mathbf{B}}^{(2)}\\ +i\tilde{\mathbf{B}}^{(2)}\end{pmatrix} e^{-i\phi} \right\} . \qquad (5)$$

Phase factors in the formula (5) is fixed between the vector and axial-vector parts of the antisymemtric tensor field for both positive- and negative- frequency solutions if one wants to have pure real fields. The $\mathbf{B}^{(3)}$ field in this case may be regarded as a part of 4-vector with respect to the Lorentz transformations.

In the present *Addendum* we are going to lift the above requirement and consider the general case:

$$\begin{pmatrix}\mathbf{B}_\perp\\ \mathbf{E}_\perp\end{pmatrix}' = \Lambda \left\{ \begin{pmatrix}\mathbf{B}^{(1)}\\ \mathbf{E}^{(1)}\end{pmatrix} e^{+i\phi} + \begin{pmatrix}\mathbf{B}^{(2)}\\ \mathbf{E}^{(2)}\end{pmatrix} e^{-i\phi} \right\} = \Lambda \left\{ \begin{pmatrix}\mathbf{B}^{(1)}\\ e^{i\alpha(x^\mu)}\mathbf{B}^{(1)}\end{pmatrix} e^{+i\phi} + \begin{pmatrix}\mathbf{B}^{(2)}\\ -e^{i\beta(x^\mu)}\mathbf{B}^{(2)}\end{pmatrix} e^{-i\phi} \right\} . \qquad (6)$$

Our formula (6) can be re-written to the formulas generalizing (6a) and (6b) of ref. [2]:

$$\mathbf{B}_i^{(1)\,\prime} = \left(1 + ie^{i\alpha}\gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij} \mathbf{B}_j^{(1)} \quad , \qquad (7a)$$

$$\mathbf{B}_i^{(2)\,\prime} = \left(1 - ie^{i\beta}\gamma(\mathbf{S}\cdot\boldsymbol{\beta}) + \frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij} \mathbf{B}_j^{(2)} \quad . \qquad (7b)$$

Then , we repeat the procedure of ref. [2] and find out that the $\mathbf{B}^{(3)}$ field may have *various* transformation laws when the transverse fields transform with the above matrix Λ. Since the Evans-Vigier field is *defined* by the formula (2) we search the transformation law for the cross product of the transverse modes $\left[\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}\right]' =?$ with taking into account (7a,7b).

$$\begin{aligned}\left[\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}\right]' &= e^{-i(\alpha-\beta)} \left[\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}\right] = \\ &= i\gamma B^{(0)} \left\{ \left[1 - \frac{e^{i\alpha}+e^{i\beta}}{2}(i\boldsymbol{\beta}\cdot\hat{\mathbf{k}})\right] (1 + \frac{\gamma^2(\boldsymbol{\beta}^2 - (\mathbf{S}\cdot\boldsymbol{\beta})^2)}{\gamma+1})_{ij}\mathbf{B}_j^{(3)} + \right. \\ &\left. + i\frac{e^{i\alpha}-e^{i\beta}}{2}(\mathbf{S}\cdot\boldsymbol{\beta})_{ij}\mathbf{B}_j^{(3)} - \gamma B^{(0)} \left[i\frac{e^{i\alpha}+e^{i\beta}}{2} + e^{i(\alpha+\beta)}(\boldsymbol{\beta}\cdot\hat{\mathbf{k}})\right]_{ij} \boldsymbol{\beta}_j \right\} . \end{aligned} \qquad (8)$$

We used above the definition $\mathbf{B}^{(3)} = B^{(0)}\hat{\mathbf{k}}$.

One can see that we recover the formula (8) of ref. [2] when the phase factors are equal to $\alpha = -\pi/2$, $\beta = -\pi/2$:[1]

[1] In the case $\alpha = +\pi/2$ and $\beta = +\pi/2$, the sign of $\boldsymbol{\beta}$ is changed to the opposite one.

$$\mathbf{B}^{(1)\,\prime}\times\mathbf{B}^{(2)\,\prime}=\mathbf{E}^{(1)\,\prime}\times\mathbf{E}^{(2)\,\prime}=i\gamma(B^{(0)})^2(1-\boldsymbol{\beta}\cdot\hat{\mathbf{k}})\left[\hat{\mathbf{k}}-\gamma\boldsymbol{\beta}+\frac{\gamma^2(\boldsymbol{\beta}\cdot\hat{\mathbf{k}})\boldsymbol{\beta}}{\gamma+1}\right]\quad . \tag{9}$$

But, we are able to obtain the transformation law as for antisymmetric tensor field, for instance when $\alpha=-\pi/2$, $\beta=+\pi/2$.[2] Namely,

$$\mathbf{B}^{(1)\,\prime}\times\mathbf{B}^{(2)\,\prime}=i\gamma\left[B^{(0)}\right]^2\left\{\hat{\mathbf{k}}-\frac{\gamma\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\hat{\mathbf{k}})}{\gamma+1}+(i\mathbf{i}\beta_y-i\mathbf{j}\beta_x)\right\}\,. \tag{10}$$

The formula (10) and the formula for opposite choice of phases lead precisely to the transformation laws of the antisymmetric tensor fields:

$$\left[\mathbf{B}^{(3)}\right]^{\prime}=\left(1\pm\gamma(\mathbf{S}\cdot\boldsymbol{\beta})+\frac{\gamma^2}{\gamma+1}(\mathbf{S}\cdot\boldsymbol{\beta})^2\right)_{ij}\mathbf{B}_j^{(3)}\quad . \tag{11}$$

$B^{(0)}$ is a true scalar in such a case.

After D. V. Ahluwalia *et al.*, ref. [14], and his commenters [12,11] (see also acknowledgements in [15]) we learnt that the theory of antisymmetric tensor field 1) admits the parity doubling; 2) suggests various relativistic equations for its description and 3) the third state of the field which in the massless limit can vanish *only* under the certain choice of normalization and frame of reference (the latter is valid when the instant form of relativistic dynamics is used).

Finally, I would like to point out that the origins of such surprising features of the antisymmetric tensor field of the second rank (unknown until recently) may be 1) possible composteness of the "photon"; 2) "hidden" electrodynamical non-locality (apart ref. [19] see also [20]); 3) "a representation space carries more information than a [particular] wave equation (e.g., Maxwell equations) – as noted also in the abstract of [19]; and 4) intrinsic interlink between gravitational and electromagnetic fields – as noted by L. de Broglie and G. Lochak (see [21]).

The question of experimental possibility of detection of the class of antisymmetric tensor fields considered in the present *Addendum* (in fact, of the *anti-hermitian modes* on using the terminology of the quantum optics) is still on schedule.

In conclusion, in my opinion, all the unpleasant incidents occured during the discussion of the $\mathbf{B}^{(3)}$ theory and related matters shows evidently serious failures of our scientific system. Finally, I want to note that the topic of the $\mathbf{B}^{(3)}$ field (in all its contradictions) is already well understood, in my opinion, and am not going to enter into these discussions any more.

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[2]In the case $\alpha=+\pi/2$ and $\beta=-\pi/2$, the sign in the third term in parentheses is changed to the opposite one.

discussions. I acknowledge many internet communications of Dr. M. Evans (1995-96) on the concept of the $\mathbf{B}^{(3)}$ field, while frequently do *not* agree with him in many particular questions. But, as shown above the postulates of the $\mathbf{B}^{(3)}$ theory may be viable.

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[1] M. W. Evans, *Physica* B**182** (1992) 227; ibid. 237; M. W. Evans, G. Hunter, S. Jeffers, S. Roy and J.-P. Vigier, *The Enigmatic Photon.* Vols. 1-4 (Kluwer Academic, Dordrecht, 1994-97).

[2] V. V. Dvoeglazov, *Found. Phys. Lett.* **10** (1997) 383 (This paper presents itself a comment on the debates between E. Comay and M. Evans and it criticizes both authors).

[3] A. Lakhtakia , *Physica* B**191** (1993) 362; D. M. Grimes, ibid. (1993) 367.

[4] E. Comay, *Chem. Phys. Lett.* **261** (1996) 601.

[5] E. Comay, *Physica* B**222** (1996) 150.

[6] E. Comay, *Found. Phys. Lett.* **10** (1997) 245.

[7] E. Comay, *Physica* A**242** (1997) 522.

[8] G. Hunter, *The* $\mathbf{B}^{(3)}$ *Field: An Assessment.* Preprint, July 1997; *The* $\mathbf{B}^{(3)}$ *Field Controversy.* Preprint, July 1997.

[9] V. V. Dvoeglazov, in *The Enigmatic Photon.* Vol. IV, eds. M. W. Evans, G. Hunter, S. Roy and J.-P. Vigier (Kluwer Academic, Dordrecht, 1997), Chapter 12.

[10] V. V. Dvoeglazov, *Apeiron* **6** (1999) 227.

[11] V. V. Dvoeglazov, *On the Importance of the Normalization.* Preprint EFUAZ FT-96-39-REV, hep-th/9712036, Dec. 1997, accepted in *Czech. J. Phys.*

[12] V. V. Dvoeglazov, *Helv. Phys. Acta* **70** (1997) 677, ibid. 686, ibid. 697.

[13] V. I. Ogievetskii and I. V. Polubarinov, *Sov. J. Nucl. Phys.* **4** (1967) 156; K. Hayashi, *Phys. Lett.* B**44** (1973) 497; M. Kalb and P. Ramond, *Phys. Rev.* D**9** (1974) 2273.

[14] D. V. Ahluwalia and D. J. Ernst, *Int. J. Mod. Phys. E***2** (1993) 397; D. V. Ahluwalia, M. B. Johnson and T. Goldman, *Phys. Lett.* B**316** (1993) 102; D. V. Ahluwalia and M. Sawicki, *Phys. Rev.* D**47** (1993) 5161.

[15] D. V. Ahluwalia, *Int. J. Mod. Phys.* A**11** (1996) 1855.

[16] "Essays on the Formal Aspects of Electromagnetic Theory" (Ed. A. Lakhtakia, World Scientific, 1993).

[17] "Advanced Electromagnetism: Foundations, Theory and Applications" (Eds. T. W. Barrett and D. M. Grimes, World Scientific, 1995).

[18] W. A. Rodrigues, jr. and J. Vaz, jr., "On the Equation $\nabla \times \mathbf{A} = \kappa \mathbf{A}$" Preprint RP-34-96, hep-th/9606128; also in "The Theory of Electron." (Ed. J. Keller and Z. Oziewicz, UNAM, 1997), *Advances in Applied Clifford Algebras* **7(C)** (1997) 457.

[19] D. V. Ahluwalia, *Mod. Phys. Lett.* A**13** (1998) 3123.

[20] A. E. Chubykalo and R. Smirnov-Rueda, *Phys. Rev.* E**53** (1996) 5373; ibid. **55** (1997) 3793; *Mod. Phys. Lett.* A**12** (1997) 1; *Int. J. Mod. Phys. A*, in press (physics/9803037).

[21] G. Lochak, *Int. J. Theor. Phys.* **24** (1985) 1019; *Ann. Fond. L. de Broglie* **20** (1995) 111; in *Advanced Electromagnetism* (ed. T. W. Barrett and D. M. Grimes), World Scientific, p.

105-147 (1995); in the forthcoming book.